Descent relations in type-0A and type-0B theories

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The type-0 theories have twice as many stable D-branes as the type II theories. In light of this added complication, we find the descent relations for D-branes in the type-0A and 0B theories. In addition, we work out how the two types of D-branes differ in their couplings to NS-NS (Neveu-Schwarz) fields.

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INTRODUCTION

In this paper, we gain further insight into type-0 D-branes by working out the descent relations for type-0 theories. Sen's descent relations in the type-II theories relate different D-branes through operations of orbifolding and tachyon kinking. These relations form an interlocking chain of relationships between the different types of D-branes. Although the type-0 theories are in many ways similar to the type-II theories, it is not immediately clear how one should draw the descent relation diagram since type-0 theories have twice the number of D-branes. This problem is addressed in Secs. III through V.

Sections I and II serve as very brief introductions to the type-0 theories and their D-brane content. In Sec. III, we review the descent relations in type-II theories and we manage to rule out certain combinations of type-0 D-branes from having any starring role in the type-0 descent relations. In Secs. IV and V, we uncover how the type-0 D-branes are related via orbifolds and kinks, respectively. By the end of Sec. V, we have pieced together the type-0 descent relations.

Section VI demonstrates the fundamental distinction between the two types of D-branes in type-0 theories. We show in Sec. VI that the two types of D-branes, D+ branes and D− branes, have opposite charges with respect to all (NS−,NS−) Neveu-Schwarz fields. We will also show how a general disk amplitude with a D+ relates to the same amplitude with a D−.

I. PERTURBATIVE SPECTRUM

Type-II superstring theories are composed of left- and right-moving pieces which reside in one of four sectors, NS± and Ramond± (R±). The + and − here denote the value of the worldsheet fermion number operator, $(-1)^F$, not to be confused with the $(-1)^{F_L^s}$ operator to be introduced later. At first blush, it appears as though there are on the order of 2^{16} possible string theories, each factor of 2 coming from whether or not a given theory contains a particular combination of sectors. Several consistency conditions pare this enormous number of possibilities to only four. Two of these are the type-IIA and -IIB theories. The other two are the less familiar type-0A and -0B theories. The consistency conditions are as follows (for a review, see [1]).

Level matching. The first condition we use to rule out some theories is the level matching condition $L_0=\tilde{L}_0$. The NS− sector has half-integer levels while the NS+, R+, and R− have integer levels. Therefore, NS− cannot be paired with any of the other three sectors.

Mutual locality. All pairs of vertex operators must be mutually local. That is, the phase obtained by taking one vertex operator in a circle around the other must be unity or else there is phase ambiguity in the amplitude.

Closed OPE. The operator product expansion (OPE) of the vertex operators in the theory must be in terms of vertex operators that are also present in the theory.

Modular invariance. Modular invariance requires that there be at least one left moving R sector and at least one right moving R sector.

The only four theories that satisfy these simple consistency requirements are the type-IIA theory,

$$(\text{NS}+,\text{NS}+)\ (\text{R}+,\text{R}-)\ (\text{NS}+,\text{R}-)\ (\text{R}+,\text{NS}+) \tag{1a}$$

the type-IIB theory,

$$(\text{NS}+,\text{NS}+)\quad (\text{R}+,\text{R}+)\quad (\text{NS}+,\text{R}+)\quad (\text{R}+,\text{NS}+) \tag{1b}$$

the type-0A theory,

$$(\text{NS}+,\text{NS}+)\quad (\text{NS-},\text{NS}-)\quad (\text{R}+,\text{R}-)\quad (\text{R}-,\text{R}+) \tag{2a}$$

and the type-0B theory,

$$(\text{NS}+,\text{NS}+)\quad (\text{NS}-,\text{NS}-)\quad (\text{R}+,\text{R}+)\quad (\text{R}-,\text{R}-). \tag{2b}$$

The perturbative spectra of the type-0 theories contain no spacetime fermions. In the NS-NS sectors, the low-lying states are the tachyon from (NS−,NS−) and the graviton, antisymmetric tensor, and dilaton from (NS+,NS+). The type-0 theories have twice as many massless R-R states as the type-II theories. In particular, type 0A has two R-R 1-forms and two R-R 3-forms; type 0B has two R-R scalars, two R-R 2-forms, and one R-R 4-form with an unconstrained 5-form field strength.

II. D-BRANES

The fact that the type-0 theories have twice as many R-R fields as the type-II theories is an indication that there may

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be twice as many stable D-branes in type 0 as compared to type II. This turns out to be correct and can be understood quite directly by examining D-branes in the boundary state formalism (for a review, see [2]). In this formalism, D-branes are represented by boundary states for the physical closed strings. These boundary states are themselves coherent closed string states.

In both the type-II and type-0 theories, there are four types of boundary states for each p,

$$|Bp,+\rangle_{\text{NS-NS}}, \quad |Bp,-\rangle_{\text{NS-NS}}, \quad |Bp,+\rangle_{\text{R-R}},$$

$$|Bp,-\rangle_{\text{R-R}}. \tag{3}$$

The $+$ and $-$ denote the boundary conditions on the world-sheet fermions and superghosts as in Eqs. (27). Linear combinations of these states must be taken to form D-brane boundary states which, in turn, must be GSO-invariant and must satisfy certain consistency conditions [2].

The D-brane boundary states in the type-0 theories are as follows:

$$\left.\begin{aligned} |Dp,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} + |Bp,+\rangle_{\text{R-R}} \\ |Dp,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} + |Bp,-\rangle_{\text{R-R}} \\ |\overline{D}p,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} - |Bp,+\rangle_{\text{R-R}} \\ |\overline{D}p,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{R-R}} \end{aligned}\right\} \text{for p even (odd) in OA (OB)} \tag{4}$$

$$\left.\begin{aligned} |\widehat{Dp},+\rangle &= |Bp,+\rangle_{\text{NS-NS}} \\ |\widehat{Dp},-\rangle &= |Bp,-\rangle_{\text{NS-NS}} \end{aligned}\right\} \text{for p odd (even) in OA (OB).} \tag{5}$$

Using η to denote ± 1, the $|Dp,\eta\rangle$ states correspond to stable D-branes. We see from the minus sign in front of the R-R boundary states that the $|\overline{D}p,\eta\rangle$ states correspond to stable anti-D-branes. The $|\widehat{Dp},\eta\rangle$ states correspond to unstable D-branes.

Let us pause for a second to make a remark on D-brane stability. The condition for stability is that the spectrum of open strings on the D-brane does not contain a tachyon. It is important not to confuse this condition with being a Bogomol'nyi-Prasad-Sommerfield (BPS) object. Of course, none of the D-branes can be BPS in the type-0 theories since there is no supersymmetry to begin with; there are no fermions in the absence of D-branes. It just so happened for D-branes in the type-II theories that the conditions of stability and BPS coincided.

It will be important for our purposes to find the spectra of open strings living on or between D-branes. The details can be found in Appendix A and the results for type-0 D-branes are given in Tables I and II. The spectra in Table I can be extrapolated to all possibilities by noting that a given spectrum is invariant under the replacements $\mathrm{D}\leftrightarrow\overline{\mathrm{D}}$ and/or $+\leftrightarrow-$. For example, from the first line of Table I, we see that the open strings beginning on a Dp$+$ and ending on a Dp$+$ are NS$+$. Therefore, the strings beginning on a $\overline{\mathrm{D}}$p$+$ and ending on a $\overline{\mathrm{D}}$p$+$ are NS$+$. Similarly, strings beginning on a Dp$-$ and ending on a Dp$-$ (or beginning on a $\overline{\mathrm{D}}$p$-$ and ending on a $\overline{\mathrm{D}}$p$-$) are also NS$+$.

We see that there are two tachyons among the open strings stretched between a $|Dp,\eta\rangle$ and a $|\overline{D}p,\eta\rangle$. One tachyon starts (at $\sigma=0$) on the $|Dp,\eta\rangle$ and ends (at $\sigma=\pi$) on the $|\overline{D}p,\eta\rangle$, and the other tachyon starts on the $|\overline{D}p,\eta\rangle$ and ends on the $|Dp,\eta\rangle$. This indicates an instability in the D$\overline{\mathrm{D}}$ pair.

We see in Table II, as expected, that there is a tachyon living on the unstable $|\widehat{Dp},\eta\rangle$ D-branes.

TABLE I. All other cases obtained by one or both of the following operations under which the spectrum is invariant: $+\leftrightarrow-$, $\mathrm{D}\leftrightarrow\overline{\mathrm{D}}$.

Open spectrum on stable D-branes (p odd in 0B, p even in 0A)		
$\sigma=0$	$\sigma=\pi$	Spectrum
Dp$+$	Dp$+$	NS$+$
Dp$+$	$\overline{\mathrm{D}}$p$+$	NS$-$
Dp$+$	Dp$-$	R$+$
Dp$+$	$\overline{\mathrm{D}}$p$-$	R$-$

TABLE II. All other cases obtained by $+\leftrightarrow-$ under which the spectrum is invariant.

Open spectrum on unstable D-branes (all p in 0A and 0B)		
$\sigma=0$	$\sigma=\pi$	Spectrum
$\widehat{\mathrm{Dp}}+$	$\widehat{\mathrm{Dp}}+$	NS$+$, NS$-$
$\widehat{\mathrm{Dp}}+$	$\widehat{\mathrm{Dp}}-$	R$+$, R$-$

III. DESCENT RELATIONS

Sen's descent relations give relations between different D-brane configurations in the type-II theories (for a review,

$$
\begin{array}{llll}
 & & \downarrow & \downarrow \\
 & \text{IIB } D(2p+1)\overline{D}(2p+1) \rightarrow & \text{IIA } D(\widehat{2p+1}) \rightarrow & \text{IIB } D(2p+1) \\
 & \downarrow & \downarrow & \\
\text{IIA } D(2p)\overline{D}(2p) \rightarrow & \text{IIB } D(\widehat{2p}) & \rightarrow \text{ IIA } D(2p) & \\
\downarrow & \downarrow & & \\
\rightarrow \text{ IIA } D(\widehat{2p-1}) \rightarrow & \text{IIB } D(2p-1) & & \\
\downarrow & & & \\
\rightarrow \text{ IIA } D(2p-2) & & &
\end{array}
$$

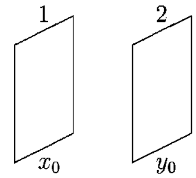
FIG. 1. Descent relations for the type-II theories. Horizontal arrows denote modding by $(-1)^{F_L^s}$. Vertical arrows denote the tachyonic kink.

see [3]). The two important operations are orbifolding by $(-1)^{F_L^s}$, where F_L^s is the spacetime fermion number of the left-movers, and kinking the tachyon field that lives on unstable configurations of D-branes. Starting with a coincident D(2p)$\overline{\text{D}}$(2p) pair in type IIA, orbifolding by $(-1)^{F_L^s}$ yields an unstable D$(\widehat{2p})$ in type IIB. Orbifolding one more time leaves us with a stable D(2p) in the type-IIA theory. Starting again with the D$(2p)\overline{\text{D}}(2p)$ pair in type IIA, but this time kinking the tachyon field that lives on the D-branes, we are left with an unstable D$(\widehat{2p\text{-}1})$ in type IIA. Kinking the remaining tachyon field gives us a stable D$(2\text{p}-2)$ in type IIA. The results are similar if we start with a D$(2p+1)\overline{\text{D}}(2\text{p}+1)$ pair in type IIB. In fact, the descent relations form an interlocking chain as shown in Fig. 1.

The natural question at this point is what the analogue of the descent relations is for the type-0 theories. Starting with a D(2p)$\overline{\text{D}}$(2p) in type 0A, we have four possibilities to consider: a choice of $+$ or $-$ for each of the two branes. Then, once we orbifold (kink), we must figure out whether we get D$(\widehat{2p})+$ or D$(\widehat{2p})-$[D$(\widehat{2p\text{-}1})+$ or D$(\widehat{2p\text{-}1})-$]. For a discussion of the differences between D$+$ and D$-$ branes, see Sec. VI.

In the type-II descent relations, every time we orbifold or kink we effectively remove one of the tachyonic degrees of freedom. A complex tachyon lives on the D$\overline{\text{D}}$ pair; orbifolding or kinking once gives an unstable D-brane with a real tachyon; orbifolding or kinking one more time gives a stable D-brane with no tachyon field. With this observation, we can quickly rule out two of the choices for the D$\overline{\text{D}}$ pair in the type-0 case. Since the open string tachyon arises from the NS$-$ sector, we see from Table I that only the Dp$+\overline{\text{D}}$p$+$ and Dp$-\overline{\text{D}}$p$-$ pairs for p odd in 0B (even in 0A) have tachyon fields living on them.

Holding out some hope for the Dp$+\overline{\text{D}}$p$-$ pair, let us see if there is any room in the type-0 descent relations for this object. Clearly, we cannot consider a tachyon kink since there is no tachyonic kink on this pair of D-branes: from Table I, we see that there are NS$+$ strings living on each of the D-branes and R$-$ strings stretched between the two. Perhaps we can orbifold this pair of D-branes by $(-1)^{F_L^s}$. However, one can take the $(-1)^{F_L^s}$ orbifold in the presence of D-branes only if that configuration of D-branes is invariant under $(-1)^{F_L^s}$. For example, in the type-II theories, $(-1)^{F_L^s}|D(2p)\rangle=|\overline{D}(2p)\rangle$ and $(-1)^{F_L^s}|\overline{D}(2p)\rangle=|D(2p)\rangle$,



$$
\begin{aligned}
1-1 &: \quad X = x_0 + \ldots \\
2-2 &: \quad X = y_0 + \ldots \\
1-2 &: \quad X = x_0 + \frac{\sigma}{\pi}(y_0 - x_0) + \ldots \\
2-1 &: \quad X = y_0 + \frac{\sigma}{\pi}(x_0 - y_0) + \ldots
\end{aligned}
$$

FIG. 2. Position zero modes corresponding to open strings in a 2 D-brane system.

so we were able to orbifold the D$\overline{\text{D}}$ pairs. Since

$$(-1)^{F_L^s}|Bp,\pm\rangle_{\text{NS-NS}}=|Bp,\pm\rangle_{\text{NS-NS}},$$
$$(-1)^{F_L^s}|Bp,\pm\rangle_{\text{R-R}}=-|Bp,\pm\rangle_{\text{R-R}}, \tag{6}$$

we see from Eq. (4) that in the type-0 theories

$$(-1)^{F_L^s}|Dp+\rangle=|\overline{D}p+\rangle,$$
$$(-1)^{F_L^s}|Dp-\rangle=|\overline{D}p-\rangle, \tag{7}$$

and

$$(-1)^{F_L^s}|\overline{D}p+\rangle=|Dp+\rangle,$$
$$(-1)^{F_L^s}|\overline{D}p-\rangle=|Dp-\rangle. \tag{8}$$

This means that the coincident Dp$+\overline{\text{D}}$p$-$ pair is not invariant under $(-1)^{F_L^s}$ and we no longer consider it as a potential participant in the type-0 descent relations. Fortunately, the Dp$+\overline{\text{D}}$p$+$ and Dp$-\overline{\text{D}}$p$-$ pairs *are* invariant under $(-1)^{F_L^s}$, so we will be able to interpret the orbifold as a projection of the open string states.

IV. $(-1)^{F_L^s}$ ORBIFOLD

Here we will consider what happens to the coincident D$(2\text{p})+\overline{\text{D}}(2\text{p})+$ pair in type 0A under the $(-1)^{F_L^s}$ orbifold. First, let us look at the spacetime bulk far from the D-branes. Locally, this is just type 0A without any open strings. Taking the orbifold of type 0A by $(-1)^{F_L^s}$ gives the type-0B theory, and vice versa (see Appendix B for details).

As we have already noted in Eqs. (7) and (8), $(-1)^{F_L^s}$ switches the D$(2\text{p})+$ and $\overline{\text{D}}(2\text{p})+$, so its action on the Chan-Paton factors is

$$\Lambda \rightarrow \sigma_1 \Lambda \sigma_1^{-1}. \tag{9}$$

Of the four Chan-Paton factors, I, σ_1, σ_2, and σ_3, only I and σ_1 are invariant under this operation. Therefore, the open strings with CP factors I and σ_1 are kept and those with CP factors σ_2 and σ_3 are thrown out.

We can see that this new object, the result of orbifolding D$(2\text{p})+\overline{\text{D}}(2\text{p})+$, is a single brane since the degrees of freedom corresponding to the relative positions of the original D-branes have been projected out. The position coordinates corresponding to their respective CP factors are as in Fig. 2.

TABLE III. R-R charges for stable D-branes.

Stable Dp R-R charges (p odd in 0B, p even in 0A)		
	q	$\bar{q}$
Dp+	1	1
$\overline{\text{D}}$p+	−1	−1
Dp−	1	−1
$\overline{\text{D}}$p−	−1	1

Writing out the lowest order degrees of freedom in terms of Chan-Paton factors, we find that we can regroup them as

$$\begin{aligned} x_0\begin{pmatrix}1&0\\0&0\end{pmatrix}+y_0\begin{pmatrix}0&0\\0&1\end{pmatrix}+\left[x_0+\frac{\sigma}{\pi}(y_0-x_0)\right]\begin{pmatrix}0&1\\0&0\end{pmatrix}\\ +\left[y_0+\frac{\sigma}{\pi}(x_0-y_0)\right]\begin{pmatrix}0&0\\1&0\end{pmatrix}\\ =\frac{1}{2}(x_0+y_0)I+\frac{1}{2}(x_0-y_0)\sigma_3+\frac{1}{2}(x_0+y_0)\sigma_1\\ +\frac{1}{2}\left[-i(x_0-y_0)+\frac{2i\sigma}{\pi}(x_0-y_0)\right]\sigma_2. \end{aligned} \tag{10}$$

The (x_0-y_0) degree of freedom multiplies only σ_2 and σ_3, which are projected out.

After orbifolding, we are left with a (2p)-brane in the type-0B theory with NS+ strings (corresponding to I) and NS− strings (corresponding to σ_1) living on it. This identifies the object as either $\widehat{\text{D(2p)}}+$ or $\widehat{\text{D(2p)}}-$. In order to distinguish between these two options, we look at the coupling of this (2p)-brane to the (NS−,NS−) tachyon and compare it to the coupling of the $\widehat{\text{D(2p)}}+$ and $\widehat{\text{D(2p)}}-$ to the (NS−,NS−) tachyon. But first we must determine what these couplings are.

We know from [5] that *stable* D-branes in the type-0 theories have the term

$$-\frac{T_p q\bar{q}}{4}\int d^{p+1}\sigma T(X) \tag{11}$$

in their low energy effective action, where T is the closed string tachyon, and q and $\bar{q}$ are the D-brane's charges under the massless R-R fields C and $\bar{C}$. The R-R charges of stable D-branes in the type-0 theories are given in Table III. Notice that $q\bar{q}=\eta$.

We know from cylinder diagrams between D-branes that the *unstable* $\widehat{\text{D}+}$ and $\widehat{\text{D-}}$ have opposite tachyon charge [6], but this cannot tell us how to assign the charges to the two types of D-branes. The solution to this can be found by comparing tachyon tadpole calculations for the stable and unstable D-branes.

The amplitude [7,8] for a stable Dp+ to emit a tachyon is

$$\begin{aligned} \langle T,k|Dp,+\rangle &=\langle T,k|(|Bp,+\rangle_{\text{NS-NS}}+|Bp,+\rangle_{\text{R-R}})\\ &=\langle T,k|Bp,+\rangle_{\text{NS-NS}}\\ &=\langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|Bp,+\rangle_{\text{NS-NS}}\\ &=\frac{T_p}{2}\langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|B_X\rangle|B_{\text{gh}}\rangle\\ &\quad\times|B_\psi,\eta\rangle_{\text{NS-NS}}|B_{\text{sgh}},\eta\rangle_{\text{NS-NS}}, \end{aligned} \tag{12}$$

where k is perpendicular to the D-brane. Now consider an unstable $\widehat{\text{D(p-1)}}+$ that is extended in p−1 of the same directions as the Dp+. The amplitude for an unstable $\widehat{\text{D(p-1)}}+$ to emit a tachyon in the same direction is

$$\begin{aligned} \langle T,k|\widehat{D(p-1)},+\rangle &=\langle T,k|B(p-1),+\rangle_{\text{NS-NS}}\\ &=\frac{T_{p-1}}{2}\langle e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}|B_X\rangle'\\ &\quad\times|B_{\text{gh}}\rangle|B_\psi,\eta\rangle'_{\text{NS-NS}}|B_{\text{sgh}},\eta\rangle_{\text{NS-NS}}. \end{aligned} \tag{13}$$

The only difference between Eqs. (12) and (13) is the normalization and the matter part of the boundary state. Both T_p and T_{p-1} are positive constants. The difference between $|B_X\rangle'$ and $|B_X\rangle$ is a minus sign on one of the X fields which does not get contracted with the $e^{ik\cdot X}$ of the tachyon since k is perpendicular to the Dp+. The difference between $|B_\psi\rangle'$ and $|B_\psi\rangle$ is a minus sign on one of the ψ fields, but none of the ψ fields in the boundary state get contracted with anything in the tachyon vertex operator. Therefore, the tachyon charge of the unstable $\widehat{\text{D(p-1)}}+$ is related to the charge of the stable Dp+ by a factor of T_{p-1}/T_p, so the tachyon tadpole term in an unstable $|\widehat{D(p-1)},\eta\rangle$ brane's low energy effective action is

$$-\frac{T_{p-1}\eta}{4}\int d^{p+1}\sigma T(X). \tag{14}$$

Note, by comparing Eqs. (11) and (14), that the Dp+ and the $\widehat{\text{D(p-1)}}+$ couple with the same sign to the closed string tachyon.

Since both the closed string tachyon and the NS-NS boundary state part of the D-branes both reside in the (NS,NS) sector which is unaffected by the orbifold, the coupling of the brane to the tachyon should be unchanged. This means that the D(2p)+$\overline{\text{D}}$(2p)+ in type 0A gets orbifolded to the $\widehat{\text{D(2p)}}+$ of type 0B.

We can understand the orbifold at the level of boundary states by considering the emission and reabsorption of closed strings by the D(2p)+$\overline{\text{D}}$(2p)+ pair. To simplify our equations, we introduce the shorthand notation

$$\langle\langle\Lambda\rangle\rangle\equiv\int dl\begin{pmatrix}|D(2p)+\rangle\\|\bar{D}(2p)+\rangle\end{pmatrix}^\dagger e^{-lH_c}\Lambda\begin{pmatrix}|D(2p)+\rangle\\|\bar{D}(2p)+\rangle\end{pmatrix}. \tag{15}$$

In this formalism, the calculation of the cylinder diagram for an open string with CP factor Λ can be rewritten as the closed string exchange amplitude $\langle\langle\Lambda\rangle\rangle$. The amplitude for a closed string to be emitted and reabsorbed by the $\mathrm{D}(2p)+\overline{\mathrm{D}}(2\mathrm{p})+$ pair is equal to

$$\left\langle\left\langle\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right\rangle\right\rangle=\langle\langle I+\sigma_1\rangle\rangle. \tag{16}$$

When we orbifold by projecting out σ_2 and σ_3, we see that this amplitude is unchanged. However, we know from our earlier discussion that the resulting object is a single D-brane. Therefore, we should be able to rewrite Eq. (16) as the emission and absorption of a closed string by a single $\mathrm{D}\widehat{(2\mathrm{p})}$. Attempting this, we find

$$\langle\langle I+\sigma_1\rangle\rangle=\begin{cases}4\int dl\langle\widehat{D(2p)}+|e^{-lH_c}|\widehat{D(2p)}+\rangle,\\[4pt] 4\int dl\langle\widehat{D(2p)}-|e^{-lH_c}|\widehat{D(2p)}-\rangle.\end{cases} \tag{17}$$

This amplitude can be written in terms of either a $\mathrm{D}\widehat{(2\mathrm{p})}+$ or a $\mathrm{D}\widehat{(2\mathrm{p})}-$, but our previous tachyon charge argument singles out the $\mathrm{D}\widehat{(2\mathrm{p})}+$.

If we orbifold one more time by $(-1)^{F_L^s}$, the bulk transforms back to type 0A. The action of the orbifold on the D-brane's open string modes can be determined by examining the two-point functions of the theory. The existence of nonzero two-point functions between open strings on the D-brane and closed strings in the bulk allows us to determine the action of $(-1)^{F_L^s}$ on the open strings by requiring the correlation functions to be invariant. As in the type-II case [3], the orbifold's effect on the $\mathrm{D}\widehat{(2\mathrm{p})}+$ is to project out the open strings with CP factor σ_1. Removing the σ_1 from Eq. (16) leaves the following amplitude for closed string emission and absorption:

$$\langle\langle I\rangle\rangle=\begin{cases}2\int dl\langle D(2p)+|e^{-lH_c}|D(2p)+\rangle\\[4pt] 2\int dl\langle\overline{D}(2p)+|e^{-lH_c}|\overline{D}(2p)+\rangle\\[4pt] 2\int dl\langle D(2p)-|e^{-lH_c}|D(2p)-\rangle\\[4pt] 2\int dl\langle\overline{D}(2p)-|e^{-lH_c}|\overline{D}(2p)-\rangle.\end{cases} \tag{18}$$

This time, the amplitude can be written in four ways, in terms of a $\mathrm{D}(2\mathrm{p})+$, $\overline{\mathrm{D}}(2\mathrm{p})+$, $\mathrm{D}(2\mathrm{p})-$, or $\overline{\mathrm{D}}(2\mathrm{p})-$. Based on the previous tachyon charge argument, we can rule out the last two possibilities, so we know the resulting object is either a stable $\mathrm{D}(2\mathrm{p})+$ or a stable $\overline{\mathrm{D}}(2\mathrm{p})+$ in type 0A. This agrees with Sen's observation in [3] that there is an inherent ambiguity as to whether the resulting object is a brane or an anti-brane.

V. TACHYONIC KINK

The other component to the descent relations is the tachyonic kink. As shown in Fig. 1, kinking one of the two tachyons on a $\mathrm{Dp}\overline{\mathrm{D}}\mathrm{p}$ in a type II theory yields a $\mathrm{D}\widehat{(\mathrm{p}\text{-}1)}$ in the same theory and kinking the remaining tachyon results in a D(p-2). This part of the descent relations is shown by taking a series of marginal deformations that connect the $\mathrm{Dp}\overline{\mathrm{D}}\mathrm{p}$ to the tachyonic kink and following what happens to the conformal field theory (CFT) under these deformations.

To outline the series of marginal deformations, we will use the $\mathrm{D1}\overline{\mathrm{D}}1$ pair in 0B for simplicity. The details of this analysis can be found in [3,4]. We begin with the $\mathrm{D1}\overline{\mathrm{D}}1$ pair wrapped on a circle of radius R and make the following deformations.

(1) We increase the gauge field on the $\overline{\mathrm{D}}1$ so that the open strings with CP factors σ_1 and σ_2 are antiperiodic around the compactification circle. In particular, the tachyon field with CP factor σ_1 is moded by half-integers as

$$T(x,t)=\sum_{n\in\mathbf{Z}}T_{n+1/2}(t)e^{i[n+(1/2)](x/R)}. \tag{19}$$

(2) The radius of the circle is taken down to $R=1/\sqrt{2}$. At this value, the $T_{\pm 1/2}$ modes are massless and, therefore, correspond to marginal deformations.

(3) A vev of $-i$ is given to $(T_{1/2}-T_{-1/2})$ which corresponds to

$$T(x)=\sin\frac{x}{2R}. \tag{20}$$

This is the tachyonic kink.

(4) The radius, R, is taken back to infinity.

Step number three will be our main focus. In order to understand the effect of this step, we first bosonize the worldsheet spinors ψ_L and ψ_R (often denoted as ψ and $\tilde{\psi}$) whose spacetime indices correspond to the compactified direction. In addition to ψ_L, ψ_R, and the corresponding X $(=X_L+X_R)$, we introduce four new spinors ξ_L, ξ_R, η_L, and η_R, and two new bosons, ϕ $(=\phi_L+\phi_R)$ and ϕ' $(=\phi_L'+\phi_R')$. The bosonization equations relating them are

$$e^{\pm i\sqrt{2}X_L}\sim(\xi_L\pm i\eta_L), \tag{21}$$

$$e^{\pm i\sqrt{2}\phi_L}\sim(\xi_L\pm i\psi_L), \tag{22}$$

$$e^{\pm i\sqrt{2}\phi_L'}\sim(\eta_L\pm i\psi_L), \tag{23}$$

and similarly for the right-moving fields. We also have the relations

$$\xi_L\eta_L\sim\partial X_L,\quad \xi_L\psi_L\sim\partial\phi_L,\quad \eta_L\psi_L\sim\partial\phi_L', \tag{24}$$

as well as the corresponding right-moving relations. Remember, these fields are specifically those fields whose spacetime indices correspond to the compactified direction. Written in terms of the new bosonic field, the tachyonic kink is made by inserting

$$\exp\left(i\frac{\sigma_1}{2\sqrt{2}} \oint \partial\phi \right) \tag{25}$$

at the boundary of the disk. In step (4), the radius is taken back to infinity by inserting vertex operators of the form $\partial X \bar{\partial} X$. When the contour integral of $\partial\phi$ is contracted around each of these operators, they are converted into $-\partial\phi' \bar{\partial}\phi'$. This corresponds to decreasing the ϕ' radius, so we must introduce a T-dual variable, ϕ'' related to the ϕ' as

$$\phi''_L = \phi'_L, \quad \phi''_R = -\phi'_R, \quad R_{\phi''} = 1/R_{\phi'}. \tag{26}$$

This converts the Neumann boundary condition on ϕ' to a Dirichlet boundary condition on ϕ'' and we are left with a D0-brane where ϕ'' is the new spacetime coordinate in place of X.

This process is easily extended to $\mathrm{Dp}\overline{\mathrm{Dp}}$ pairs for p other than 1 since the other worldsheet fields are left unchanged. This is, in fact, the key to understanding whether a $\mathrm{Dp}+\overline{\mathrm{Dp}}+$ gets kinked to a $\widehat{\mathrm{D(p\text{-}1)}}+$ or a $\widehat{\mathrm{D(p\text{-}1)}}-$. Let us take a look now at what the $+$ and $-$ correspond to in terms of worldsheet fields. The boundary state $|Dp,\eta\rangle$ satisfies the following equations:

$$\begin{aligned}
&\partial_n X^\mu |Dp,\eta\rangle = 0, \quad \mu = 0,\ldots,p\\
&(X^i - y^i)|Dp,\eta\rangle = 0, \quad i = p+1,\ldots,9\\
&(\psi^\mu - \eta\tilde{\psi}^\mu)|Dp,\eta\rangle = 0, \quad \mu = 0,\ldots,p\\
&(\psi^i + \eta\tilde{\psi}^i)|Dp,\eta\rangle = 0, \quad i = p+1,\ldots,9\\
&(b - \tilde{b})|Dp,\eta\rangle = 0,\\
&(c - \tilde{c})|Dp,\eta\rangle = 0,\\
&(\gamma - \eta\tilde{\gamma})|Dp,\eta\rangle = 0,\\
&(\beta - \eta\tilde{\beta})|Dp,\eta\rangle = 0.
\end{aligned} \tag{27}$$

The first four of these equations are the familiar boundary conditions on the matter fields. The last four can be obtained by demanding Becchi-Rouet-Stora-Tyutin (BRST) invariance of the boundary state [8].

The only worldsheet fields that are affected by the kink are those whose spacetime index is the same as the compactified direction. For example, no matter what tachyonic kinking procedure we can imagine, ψ^0 will certainly be unaffected. Since the η value of the $|Dp,\eta\rangle$ D-brane can be read off from the boundary condition on ψ^0, η is invariant under all marginal deformations corresponding to tachyonic kinks. This means that a $\mathrm{Dp}+\overline{\mathrm{Dp}}+$ gets kinked to a $\widehat{\mathrm{D(p\text{-}1)}}+$.

TABLE IV. The other two cases obtained by the following operation under which the spectrum is invariant: $\mathrm{D} \leftrightarrow \overline{\mathrm{D}}$.

Open spectrum on stable D-branes (p odd in IIB, p even in IIA)		
$\sigma=0$	$\sigma=\pi$	Spectrum
Dp	Dp	NS+, R−
Dp	$\overline{\mathrm{Dp}}$	NS−, R+

Now we claim that the rest of the kink analysis goes through the same as it did in the case of the type II theories. How can we be so sure of this? The type-0 and type-II theories differ in their perturbative closed string spectra, but the marginal deformations needed to bring about a tachyonic kink uses only those parts of the closed string spectra that type 0 and type II have in common. In particular, the only closed string VEV that is deformed is that of the graviton which can be found in the (NS+,NS+) sector of all type-0 and type-II theories. All other deformations have to do with open strings, and the bosonic open string spectra on D-branes in type-0 and type-II theories are identical. This can be seen by comparing Tables I and II with Tables IV and V in Appendix A.

Let us check that the $\mathrm{Dp}+\overline{\mathrm{Dp}}+$ gets kinked to the $\widehat{\mathrm{D(p\text{-}1)}}+$ by considering the amplitude for the emission of a closed string tachyon. From Table III and Eq. (11), we see that the combined $\mathrm{D1}+\overline{\mathrm{D1}}+$ pair in type 0B has a nonzero tachyon charge. (Recall that $\eta = q\bar{q}$.) The amplitude under consideration is the closed tachyon tadpole amplitude: a disk with the tachyon vertex operator inserted in the bulk. Again, kinematics force the momentum of the emitted tachyon to be perpendicular to the $\mathrm{D1}+\overline{\mathrm{D1}}+$ pair. Therefore, there are no potential contractions between the tachyon vertex operator, $e^{-\Phi-\tilde{\Phi}}e^{-ik\cdot X}$, and the tachyonic kink operator in Eq. (25). The sign of the amplitude is not changed by the marginal deformations, so the result is a $\widehat{\mathrm{D0}}$ brane that couples to the closed tachyon with the same sign as the $\mathrm{D1}+\overline{\mathrm{D1}}+$, namely a $\widehat{\mathrm{D0}}+$.

The result we have established here for the $\mathrm{D1}+\overline{\mathrm{D1}}+$ pair in type 0B can easily be extended to all $\mathrm{Dp}+\overline{\mathrm{Dp}}+$ pairs and $\mathrm{Dp}-\overline{\mathrm{Dp}}-$ pairs for p even in 0A and p odd in 0B. The tachyonic kink on an unstable $\widehat{\mathrm{Dp}}+$ or $\widehat{\mathrm{Dp}}-$, for p>0, can be analyzed by the following procedure [3]. Take the unstable $\widehat{\mathrm{D1}}+$ in 0A as an example. If we T-dualize the $\mathrm{D1}+\overline{\mathrm{D1}}+$ pair in type 0B, we find that the $\mathrm{D0}+\overline{\mathrm{D0}}+$ pair in 0A is connected by marginal deformations to the $\widehat{\mathrm{D1}}+$ in

TABLE V. Open strings on unstable D-branes.

Open spectrum on unstable D-branes (all p in IIA and IIB)		
$\sigma=0$	$\sigma=\pi$	Spectrum
$\widehat{\mathrm{Dp}}$	$\widehat{\mathrm{Dp}}$	NS+, NS−, R+, R−

$$
\begin{array}{llll}
 & & \downarrow & \downarrow \\
 & \text{0B D(2p+1)+}\overline{\text{D}}\text{(2p+1)+} \rightarrow & \text{0A }\widehat{\text{D(2p+1)}}\text{+} \rightarrow & \text{0B D(2p+1)+} \\
 & \downarrow & \downarrow & \\
\text{0A D(2p)+}\overline{\text{D}}\text{(2p)+} \rightarrow & \text{0B }\widehat{\text{D(2p)}}\text{+} & \rightarrow \text{0A D(2p)+} & \\
\downarrow & \downarrow & & \\
\rightarrow \text{0A }\widehat{\text{D(2p-1)}}\text{+} \rightarrow & \text{0B D(2p-1)+} & & \\
\downarrow & & & \\
\rightarrow \text{0A D(2p-2)+} & & &
\end{array}
$$

FIG. 3. Descent relations for the type-0 theories. Horizontal arrows denote modding by $(-1)^{F_L^s}$. Vertical arrows denote the tachyonic kink. A similar diagram exists with $+\rightarrow-$.

0A. By running the marginal deformations backwards, we see that the D0+$\overline{\text{D}}$0+ corresponds to a kink-antikink pair on the $\widehat{\text{D1}}$+. This allows us to identify the tachyonic kink on the $\widehat{\text{D1}}$+ as a stable D0+ in type 0A. The flowchart of descent relations in the type-0 theories is given in Fig. 3.

VI. $|Dp,\eta\rangle$: $\eta=+1$ VS $\eta=-1$

It is important to stress that the value of η in $|Dp,\eta\rangle$ does not just affect the R-R charges of the D-brane. It has an important effect on many string amplitudes. In fact, we will be able to show below that Dp+ and Dp− branes have the same tadpole couplings to all (NS+,NS+) fields and opposite tadpole couplings to all (NS−,NS−) fields.

Let us first try to see the opposite tachyon charges of the Dp+ and Dp− at the level of a string calculation. Emission of a tachyon from a D-brane in a type-0 theory is given by a disk amplitude with the tachyon vertex operator in the bulk and appropriate boundary conditions on the edge. Note from Eq. (27) that these boundary conditions depend on η. Equations (27) are in terms of the fields defined on the upper-half plane, so once we map our tachyon amplitude to the upper half plane, the following η-dependent equations must hold on the real axis:

$$\tilde{\psi}^\mu=\eta\psi^\mu,\quad \tilde{\psi}^i=-\eta\psi^i, \tag{28}$$

$$\tilde{\gamma}=\eta\gamma,\quad \tilde{\beta}=\eta\beta. \tag{29}$$

The doubling trick [9] extends the string calculation to the entire complex plane by defining

$$\tilde{\psi}^\mu(\bar{z})=\eta\psi^\mu(\bar{z}),\quad \tilde{\psi}^i(\bar{z})=-\eta\psi^i(\bar{z}), \tag{30}$$

$$\tilde{\gamma}(\bar{z})=\eta\gamma(\bar{z}),\quad \tilde{\beta}(\bar{z})=\eta\beta(\bar{z}) \tag{31}$$

on the lower half plane. In actual calculations, β and γ are rebosonized in terms of the free bosons Φ and χ as

$$\beta\cong e^{-\Phi+\chi}\partial\chi,\quad \gamma\cong e^{\Phi-\chi}. \tag{32}$$

The doubling trick identifications on γ and β can be rewritten as

$$\tilde{\Phi}(\bar{z})=\Phi(\bar{z})+\frac{i\pi}{2}(1-\eta),$$

$$\tilde{\chi}(\bar{z})=\chi(\bar{z}). \tag{33}$$

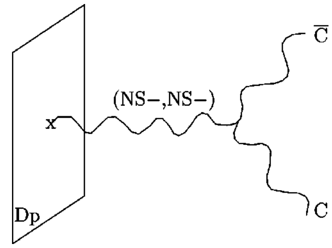


FIG. 4. $C\rightarrow\bar{C}$ scattering off a Dp-brane.

After mapping to the upper half plane and then using the doubling trick, the amplitude has become

$$\langle e^{-\Phi(z)-\Phi(\bar{z})-i\pi(1-\eta)/2}e^{-ik\cdot X}\rangle$$
$$=(-1)^{(1-\eta)/2}\langle e^{-\Phi(z)-\Phi(\bar{z})}e^{-ik\cdot X}\rangle. \tag{34}$$

Here we see the explicit dependence on η of the D-brane's tachyon charge.

A somewhat complicated, but instructive, example is to look at C to $\bar{C}$ scattering as depicted in Fig. 4, where C and $\bar{C}$ are massless bosons from the two different R-R sectors.

The D-brane in type-0 theories couples to (NS−,NS−) closed strings and there are vertices in the low energy spacetime action that connect (NS−,NS−) strings to a C and a $\bar{C}$ [5]. The string diagram that contributes to this process is a disk with V_C and $V_{\bar{C}}$ operators. These massless R-R vertex operators are given by

$$V_i^{C_{m-1}}(z_i,\bar{z}_i)=(P_-\Gamma_{i(m)})^{AB}:V_{-1/2\,A}(p_i,z_i):$$
$$\times:\tilde{V}_{-1/2B}(p_i,\tilde{z}_i):, \tag{35}$$

$$V_i^{\bar{C}_{m-1}}(z_i,\bar{z}_i)=(P_+\Gamma_{i(m)})^{AB}:V_{-1/2\,A}(p_i,z_i):$$
$$\times:\tilde{V}_{-1/2B}(p_i,\tilde{z}_i):, \tag{36}$$

where we are using the notation of [10]. The objects in these vertex operators are defined as

$$V_{-1/2A}(p_i,z_i)=e^{-\Phi(z_i)/2}S_A(z_i)e^{ip_i\cdot X_L(z_i)}, \tag{37}$$

$$P_\pm=(1\pm\gamma_{11})/2, \tag{38}$$

$$\Gamma_{(n)}=\frac{a_n}{n!}F_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n}, \tag{39}$$

where S_A is the spin field, $\gamma_{11}=\gamma^0\ldots\gamma^9$, and $F_n=dC_{n-1}$.

Under the doubling trick, the spin field $\tilde{S}_A$ will be identified as

$$\tilde{S}_A(\bar{z})=M_A{}^B S_B(\bar{z}), \tag{40}$$

for some matrix M. This matrix can be specified [10] by considering the following OPE's:

$$\psi^\mu(z)S_A(w)\sim(z-w)^{-1/2}\frac{1}{\sqrt{2}}(\gamma^\mu)_A{}^B S_B(w)+\ldots \tag{41}$$

$$\tilde{\psi}^\mu(\bar{z})\tilde{S}_A(\bar{w})\sim(\bar{z}-\bar{w})^{-1/2}\frac{1}{\sqrt{2}}(\gamma^\mu)_A{}^B\tilde{S}_B(\bar{w})+\ldots. \tag{42}$$

The doubling trick identification for $\tilde{\psi}^\mu$ is $\tilde{\psi}^\mu(\bar{z})=\eta D^\mu{}_\nu\psi^\nu(\bar{z})$, where $D^\mu{}_\nu=(\delta^\alpha{}_\beta,-\delta^i{}_j)$. In order for Eq. (42) to be consistent with Eq. (41), M must satisfy

$$(\gamma^\mu)_A{}^B=D^\mu{}_\nu(M^{-1}\gamma^\nu M)_A{}^B. \tag{43}$$

This can be rewritten as $(M\gamma^\mu)=D^\mu{}_\nu(\gamma^\nu M)$ which implies that M is of the form

$$M=\begin{cases} a\gamma^0\ldots\gamma^p & \text{for p+1 odd, } \eta=1,\\ b\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 even, } \eta=1,\\ c\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 odd, } \eta=-1,\\ d\gamma^0\ldots\gamma^p & \text{for p+1 even, } \eta=-1.\end{cases} \tag{44}$$

To fix the phases, the OPE's

$$S_A(z)S_B(w)\sim(z-w)^{-5/4}C^{-1}_{AB}+\ldots \tag{45}$$

$$\tilde{S}_A(\bar{z})\tilde{S}_B(\bar{w})\sim(\bar{z}-\bar{w})^{-5/4}C^{-1}_{AB}+\ldots \tag{46}$$

are used to find that $M^{-1}=C^{-1}M^TC$. Since all the γ^μ and γ_{11} anticommute with C, we find the phases up to an overall sign:

$$M=\begin{cases} \pm i\gamma^0\ldots\gamma^p & \text{for p+1 odd, } \eta=1\\ \pm\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 even, } \eta=1\\ \pm\gamma^0\ldots\gamma^p\gamma_{11} & \text{for p+1 odd, } \eta=-1\\ \pm i\gamma^0\ldots\gamma^p & \text{for p+1 even, } \eta=-1.\end{cases} \tag{47}$$

From now on, we will write M as M_η to distinguish between the two forms it takes for fixed p. Equation (47) gives the relationships between M_+ and M_- as

$$M_-=\pm iM_+\gamma_{11}. \tag{48}$$

The amplitude for $C\to\bar{C}$ scattering off a Dp+ is [11]

$$\begin{aligned}A(C,\bar{C})_+=-\frac{i\kappa^2T_p}{2}\bigg[&\frac{1}{2}\mathrm{Tr}(P_-\Gamma_{1(m)}M_+\gamma^\mu)\\ &\times\mathrm{Tr}(P_+\Gamma_{2(n)}M_+\gamma_\mu)B(-t/2+1/2,-2s)\\ &-\mathrm{Tr}(P_-\Gamma_{1(m)}C^{-1}\Gamma^T_{2(n)}C)\\ &\times B(-t/2-1/2,-2s+1)\\ &-\mathrm{Tr}(P_-\Gamma_{1(m)}M_+\Gamma_{2(n)}M_+)\\ &\times B(-t/2+1/2,-2s+1)\bigg].\end{aligned} \tag{49}$$

Since the Euler beta function is defined as

$$B(a,b)=\int_0^1 dy\, y^{a-1}(1-y)^{b-1}, \tag{50}$$

we see that the poles in the t channel are $m^2=(4n-2)/\alpha'$ for $n=0,1,\ldots$. These poles correspond to the masses of the closed strings in the (NS−,NS−) sector.

To obtain $A(C,\bar{C})_-$, the amplitude for $C\to\bar{C}$ scattering off a Dp−, from $A(C,\bar{C})_+$, we must replace M_+ with M_- and $e^{-\Phi(\bar{z})/2}$ with $e^{-\Phi(\bar{z})/2-i\pi/2}$. It is simple to check that the amplitude is invariant under replacing M_+ with M_-. In the correlation function, there are two factors of $e^{-\Phi(\bar{z})/2}$ coming from the two R-R vertex operators. After replacing them with $e^{-\Phi(\bar{z})/2-i\pi/2}$, each one contributes a factor of i for a total phase of -1. In summary, we find that

$$A(C,\bar{C})_+=-A(C,\bar{C})_-. \tag{51}$$

This shows that the Dp+ and Dp− couple with opposite signs to all (NS−,NS−) fields.

How can this phenomenon be understood in a direct manner? Consider the tadpole amplitude for emission of a closed string from a D-brane. If the closed string is in one of the NS-NS sectors, the amplitude is a disk with the closed string vertex operator in the $(-1,-1)$ picture. For a NS-NS string, the amplitude for emission from a Dp+ can be converted into an amplitude for emission from a Dp− by multiplying by -1 for each factor of $e^{-\tilde{\Phi}}$ and $\tilde{\psi}^\mu$. In the $(-1,-1)$ picture, the NS-NS vertex operator has as many $\tilde{\psi}$'s as does the corresponding Fock state. Therefore, the Dp− amplitude differs from the Dp+ amplitude by a factor of $(-1)^{\tilde{F}}$, where $\tilde{F}$ is the right-moving worldsheet fermion number of the NS-NS closed string state. In other words, Dp+ and Dp− have the same tadpole couplings to all (NS+,NS+) fields and opposite tadpole couplings to all (NS−,NS−) fields.

It is clear how to generalize this to a general disk amplitude on a D-brane. To convert a general disk amplitude for a D+ into the same amplitude with a D−, we multiply by -1 for each $e^{-\tilde{\Phi}}$ and $\tilde{\psi}^\mu$, and we replace M_+ with $\pm iM_+\gamma_{11}$ for each spin field. Since a fermionic state cannot transform into a bosonic one, the number of M_η's will be even in any nonzero amplitude, so the sign ambiguity in that replacement is insignificant.

VII. SUMMARY

We set out to find the descent relations for the type-0 theories. We found that we must start with either a D+$\overline{\text{D}}$+ pair or a D−$\overline{\text{D}}$− pair and that the + and − are invariant under the orbifold and kink operations. This means we have two copies of the usual descent relation chain for the type-0 theories: one for D+ branes and one for D− branes. We then asked why we should care about the distinction between a D+ brane and a D− brane. While it is fairly well known that the stable D+ and D− have the same coupling to half of the massless R-R fields and equal and opposite couplings to the other half, we have shown that the D+ and D− have the same tadpole couplings to half of the NS-NS fields and equal and opposite tadpole couplings to the other half.

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APPENDIX A: OPEN STRING SPECTRUM

In this appendix we will find the open string spectrum on type-II and type-0 D-branes. We begin by considering the closed string exchange amplitudes between boundary states, which are given in [12]. Motivated by the usual worldsheet duality of the cylinder diagram, this result can be converted into an open string loop amplitude. The results are as follows:

$$\begin{aligned}
&\int dl \;\; {}_{\text{NS-NS}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{NS-NS}} = \int \frac{dt}{2t}\text{Tr}_{\text{NS}}[e^{-tH_{\text{open}}}]\\
&\int dl \;\; {}_{\text{NS-NS}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,-\eta\rangle_{\text{NS-NS}} = -\int \frac{dt}{2t}\text{Tr}_{\text{R}}[e^{-tH_{\text{open}}}]
\end{aligned} \tag{A1}$$

$$\begin{aligned}
&\int dl \;\; {}_{\text{R-R}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{R-R}} = \int \frac{dt}{2t}\text{Tr}_{\text{NS}}[(-1)^F e^{-tH_{\text{open}}}]\\
&\int dl \;\; {}_{\text{R-R}}\langle Bp,\eta|e^{-lH_{\text{closed}}}|Bp,-\eta\rangle_{\text{R-R}} = -\int \frac{dt}{2t}\text{Tr}_{\text{R}}[(-1)^F e^{-tH_{\text{open}}}].
\end{aligned}$$

We will combine Eqs. (A1) with the expressions [2] for the type-II D-branes in terms of boundary states,

$$\left.\begin{aligned}
|Dp\rangle &= (|Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}) + (|Bp,+\rangle_{\text{R-R}} + |Bp,-\rangle_{\text{R-R}})\\
|\overline{Dp}\rangle &= (|Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}) - (|Bp,+\rangle_{\text{R-R}} + |Bp,-\rangle_{\text{R-R}})
\end{aligned}\right\} \text{for p even (odd) in IIA (IIB)} \tag{A2}$$

$$|\widehat{Dp}\rangle = |Bp,+\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{NS-NS}}\} \text{ for all p in IIA and IIB} \tag{A3}$$

and the expressions for the type-0 D-branes in terms of boundary states,

$$\left.\begin{aligned}
|Dp,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} + |Bp,+\rangle_{\text{R-R}}\\
|Dp,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} + |Bp,-\rangle_{\text{R-R}}\\
|\overline{D}p,+\rangle &= |Bp,+\rangle_{\text{NS-NS}} - |Bp,+\rangle_{\text{R-R}}\\
|\overline{D}p,-\rangle &= |Bp,-\rangle_{\text{NS-NS}} - |Bp,-\rangle_{\text{R-R}}
\end{aligned}\right\} \text{for p even (odd) in 0A (0B)} \tag{A4}$$

$$\left.\begin{aligned}
|\widehat{Dp},+\rangle &= |Bp,+\rangle_{\text{NS-NS}}\\
|\widehat{Dp},-\rangle &= |Bp,-\rangle_{\text{NS-NS}}
\end{aligned}\right\} \text{for all p in 0A and 0B.} \tag{A5}$$

It is impossible for a R-R string to spontaneously convert into a NS-NS string, or vice versa, so we know that

$${}_{\text{NS-NS}}\langle Bp,\eta'|e^{-lH_{\text{closed}}}|Bp,\eta\rangle_{\text{R-R}} = 0. \tag{A6}$$

Now, to find the spectrum on open strings beginning and ending on a stable Dp+ in the type-0 theories, we will rewrite the closed string exchange diagram as a trace over open string states. We have everything we need to perform this calculation; combining Eqs. (A1) and (A4), we find

$$\begin{aligned}\int & dl\langle Dp,+|e^{-lH_{\text{closed}}}|Dp,+\rangle \\ &= \int dl \quad {}_{\text{NS-NS}}\langle Bp,+|e^{-lH_{\text{closed}}}|Bp,+\rangle_{\text{NS-NS}} \\ &\quad + \int dl \quad {}_{\text{R-R}}\langle Bp,+|e^{-lH_{\text{closed}}}|Bp,+\rangle_{\text{R-R}} \\ &= \int \frac{dt}{2t}\text{Tr}_{\text{NS}}[e^{-tH_{\text{open}}}] \\ &\quad + \int \frac{dt}{2t}\text{Tr}_{\text{NS}}[(-1)^F e^{-tH_{\text{open}}}] \\ &= \int \frac{dt}{2t}\text{Tr}_{\text{NS}}[(1+(-1)^F)e^{-tH_{\text{open}}}] \\ &= \int \frac{dt}{t}\text{Tr}_{\text{NS}+}[e^{-tH_{\text{open}}}]. \qquad \text{(A7)}\end{aligned}$$

So we see that the open strings beginning and ending on a stable Dp+ in the type-0 theories are NS+. Proceeding in this manner, we can find the spectrum of open strings on all possible combinations of D-branes in the type-0 and type-II theories. The full results for the type-0 theories are given in Tables I and II in Sec. II. The results for the type-II theories are given in Tables IV and V.

APPENDIX B: ORBIFOLD OF 0A/0B

The action of $(-1)^{F_L^s}$ can be represented as a 2π spacetime rotation on the left movers. Under this rotation, the left-sector bosons (NS) are invariant and the left-sector fermions (R) pick up a minus sign. We can pick any spatial plane for this rotation and for our purposes here we select the 8-9 plane.

The situation is greatly simplified if we use complexified coordinates [1] for those left-moving fields whose indices are in the 8-9 plane,

$$\Psi^4 = \frac{1}{\sqrt{2}}(\psi^8 + i\psi^9),$$

$$\Psi^{\bar{4}} = \frac{1}{\sqrt{2}}(\psi^8 - i\psi^9), \qquad \text{(B1)}$$

$$\partial Z^4 = \frac{1}{\sqrt{2}}(\partial X^8 + i\partial X^9),$$

$$\partial Z^{\bar{4}} = \frac{1}{\sqrt{2}}(\partial X^8 - i\partial X^9). \qquad \text{(B2)}$$

With this notation, a rotation on the left-movers by angle θ in the 8-9 plane has the following action on the fields:

$$\Psi^4 \to e^{i\theta}\Psi^4,$$

$$\Psi^{\bar{4}} \to e^{-i\theta}\Psi^{\bar{4}}, \qquad \text{(B3)}$$

$$\partial Z^4 \to e^{i\theta}\partial Z^4,$$

$$\partial Z^{\bar{4}} \to e^{-i\theta}\partial Z^{\bar{4}}. \qquad \text{(B4)}$$

We wish to find the orbifold of type 0A by $(-1)^{F_L^s}$. This is an asymmetric, Abelian orbifold with group elements $\{1,(-1)^{F_L^s}\}$. The untwisted sector, corresponding to the identity element, is simply the projection of 0A on states invariant under $(-1)^{F_L^s}$. It is clear that the invariant states are those in the sectors (NS+,NS+) and (NS−,NS−). Let us check that we get the same result by representing $(-1)^{F_L^s}$ as a rotation by 2π on the left-movers. On the NS sector ground state vertex operator, $1\to 1$; the NS sector is invariant. To consider the action on the R sector ground state vertex operator, we must bosonize the complexified fermions as

$$\Psi^4 = e^{iH^4},$$

$$\Psi^{\bar{4}} = e^{-iH^4}, \qquad \text{(B5)}$$

and likewise for the other fermions. In terms of these bosonic H fields, the spin operator takes the form

$$\Theta_s = e^{i\sum_{a=1}^{4} s_a H^a}, \qquad \text{(B6)}$$

where the $s_a = \pm 1/2$. Since Ψ^4 transforms under the $\theta = 2\pi$ rotation as (B3), $\exp(\frac{1}{2}iH^4)$ transforms as

$$e^{(1/2)iH^4} \to e^{i\pi}e^{(1/2)iH^4} = -e^{(1/2)iH^4}. \qquad \text{(B7)}$$

Therefore, the spin field, and subsequently the left-moving R sector vertex operator, picks up a minus sign from the 2π rotation; the (R+,R−) and (R−,R+) sectors are projected out.

In the twisted sector, the boundary conditions on the ∂Z^4 and Ψ^4 fields are as follows:

$$\partial Z^4(\sigma+2\pi) = e^{2\pi i}\partial Z^4(\sigma),$$

$$\partial Z^{\bar{4}}(\sigma+2\pi) = e^{-2\pi i}\partial Z^{\bar{4}}(\sigma), \qquad \text{(B8)}$$

$$\Psi^4(\sigma+2\pi) = e^{2\pi i(\beta+\nu)}\Psi^4(\sigma),$$

$$\Psi^{\bar{4}}(\sigma+2\pi) = e^{-2\pi i(\beta+\nu)}\Psi^{\bar{4}}(\sigma), \qquad \text{(B9)}$$

where $\nu=0$ for R, $\nu=1/2$ for NS, and $\beta=1$. At first glance, it appears as though the boundary conditions are unchanged. However, if we continuously change the boundary condition factor $\exp(2\pi i\beta)$ from $\beta=0$ to $\beta=1$, we see that the moding of the Fourier coefficients has changed from n for both ∂Z^4 and $\partial Z^{\bar{4}}$ and $n+\nu$ for both Ψ^4 and $\Psi^{\bar{4}}$ to

$$\begin{aligned} \alpha^4&:\ n+1,\\ \alpha^{\bar{4}}&:\ n-1,\\ \Psi^4&:\ n+1+\nu,\\ \Psi^{\bar{4}}&:\ n-1-\nu. \end{aligned} \tag{B10}$$

This phenomenon, known as spectral flow, has an important consequence for the ground state of the theory. When we began with $\beta=0$, the ground state was defined as

$$\Psi^4_{n+\nu}|0\rangle=\Psi^{\bar{4}}_{n+1-\nu}|0\rangle=0 \quad \text{for } n=0,1,\ldots, \tag{B11}$$

with similar equations for the other Ψ. The effect of continuously changing β from 0 to 1 is that we replace ν with $\nu+1$ in these equations. The ground state now satisfies the conditions

$$\Psi^4_{n+\nu+1}|0\rangle=\Psi^{\bar{4}}_{n-\nu}|0\rangle=0 \quad \text{for } n=0,1,\ldots. \tag{B12}$$

The $|0\rangle$ state is no longer the ground state because $\Psi^4_\nu|0\rangle\neq 0$ and $\Psi^{\bar{4}}_{-\nu}|0\rangle=0$. The true ground state is

$$|0\rangle'=\Psi^4_\nu|0\rangle \tag{B13}$$

since

$$\Psi^4_\nu|0\rangle'=\Psi^4_\nu\Psi^4_\nu|0\rangle=0 \tag{B14}$$

and

$$\Psi^{\bar{4}}_{-\nu}|0\rangle'=\Psi^{\bar{4}}_{-\nu}\Psi^4_\nu|0\rangle=\{\Psi^{\bar{4}}_{-\nu},\Psi^4_\nu\}|0\rangle=|0\rangle\neq 0. \tag{B15}$$

However, now the GSO condition on the left-movers,

$$(-1)^F|0\rangle=\pm|0\rangle \tag{B16}$$

has become

$$(-1)^F|0\rangle'=-\Psi^4_\nu(-1)^F|0\rangle=\mp|0\rangle'. \tag{B17}$$

We see that the GSO conditions on the left-movers has been reversed.

This leaves us with the following twisted sector:

$$(\mathrm{NS}-,\mathrm{NS}+)\quad(\mathrm{NS}+,\mathrm{NS}-)\quad(\mathrm{R}-,\mathrm{R}-)\quad(\mathrm{R}+,\mathrm{R}+). \tag{B18}$$

Of these four groups of states, we keep only those that will combine with the untwisted sector to give us a modular invariant theory. For Abelian orbifolds, the correct criteria for the twisted states to ensure modular invariance is level matching. In the (NS−,NS+) and (NS+,NS−) sectors, there is no way to obtain $L_0=\tilde{L}_0$, so we drop these states.

In the end, we are left with the (NS+,NS+) and (NS−,NS−) states from the untwisted sector and the (R−,R−) and (R+,R+) states from the twisted sector. Combined, these give the spectrum of the type-0B theory as given in Eq. (2b). The argument works in the same way to get type 0A from 0B.

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