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AdS solutions to the 2D type 0A effective action

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We present a two-parameter family of AdS solutions to the two-dimensional type 0A effective action.

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I. INTRODUCTION

AdS backgrounds of string theory are often fruitful arenas for studying holographic dualities and for constructing sigma models with R-R fluxes, among other things. AdS backgrounds of type 0A string theory should be no exception. In particular, the recent discovery of a matrix quantum mechanics dual to two-dimensional type 0A string theory [1,2] suggests a promising direction for understanding AdS_2/CFT_1 [3]. Also, two-dimensional AdS presents one of the simplest backgrounds in which to study sigma models in R-R flux.

As a first step in these pursuits, we present here a two-parameter family of AdS_2 solutions to the two-dimensional type 0A effective action. One of the parameters is the tachyon expectation value T (equivalently, the ratio of dualized R-R field strengths, q_+^2/q_-^2). The other parameter is the string coupling $e^{2\Phi}$ (equivalently, the magnitude of the field strength q^2). In these solutions, string loops can be suppressed by reducing the string coupling, but the high curvature of the spaces makes higher order α' corrections important.

In Sec. II, we briefly review the 2D 0A string theory. In Sec. III, we present our family of AdS_2 solutions. In Sec. IV, we discuss possible corrections to our solutions from higher order terms in the effective action.

II. TWO-DIMENSIONAL TYPE 0A

The ten-dimensional type 0A string theory is given by the same world sheet action as the type IIA string, but with a Gliozzi-Scherk-Olive projection onto the closed string sectors

$$(\mathrm{NS}+,\mathrm{NS}+)\quad(\mathrm{NS}-,\mathrm{NS}-)\quad(\mathrm{R}+,\mathrm{R}-)\quad(\mathrm{R}-,\mathrm{R}+), \tag{2.1}$$

where $+$ and $-$ denote the eigenvalue of the world sheet fermion number operator $(-1)^F$. In ten dimensions, each of these sectors contains a tower of states corresponding to the possible transverse oscillations. In two dimensions, however, there is no room for transverse oscillations, so the situation is much simpler. We have the graviton $g_{\mu\nu}$ and dilaton Φ in the (NS+, NS+) sector, the tachyon T in the (NS−, NS−) sector, and two gauge fields $C_\mu^{(\pm)}$ from the R-R sectors that give rise to two field strengths, $F_{\mu\nu}^{(\pm)}$. An allowed background for this theory is the two-dimensional linear dilaton vacuum ($g_{\mu\nu}=\eta_{\mu\nu}$ and $\Phi=\sqrt{\frac{2}{\alpha'}}\phi$) with an exponential tachyon wall [$T=\mu e^{(2/\alpha')^{1/2}\phi}$] and zero field strengths ($F_{\mu\nu}^{(\pm)}=0$). The world sheet action for this string theory is the action of $\mathcal{N}=1$ super-Liouville theory plus the action for a free scalar superfield.

The action for $\mathcal{N}=1$ super-Liouville theory can be written in superfield formalism[1] as

$$S_{\mathrm{SLT}}=\frac{1}{4\pi}\int d^2zd^2\theta(D\Phi\overline{D}\Phi+2i\mu e^{b\Phi}), \tag{2.2}$$

where Φ is the scalar superfield

$$\Phi=\sqrt{\frac{2}{\alpha'}}\phi+i\theta\psi+i\overline{\theta}\,\overline{\psi}+i\theta\overline{\theta}F, \tag{2.3}$$

and the covariant derivatives are given by

$$D=\partial_\theta+\theta\partial_z,\qquad \overline{D}=\partial_{\overline{\theta}}+\overline{\theta}\partial_{\overline{z}}. \tag{2.4}$$

In the case $b=1$, this yields a theory with central charge $\hat{c}=9$. When combined with the $\hat{c}=1$ theory of a free scalar superfield X with action

$$S_{\mathrm{X}}=\frac{1}{4\pi}\int d^2zd^2\theta(DX\overline{D}X), \tag{2.5}$$

we get a critical superconformal field theory with central charge $\hat{c}=10$. This is the world sheet action for two-dimensional type 0A in the linear dilaton vacuum.

The effective spacetime action was calculated in [4] and was found to be

$$\int d^2x\sqrt{-g}\Big\{\frac{e^{-2\Phi}}{2\kappa^2}\Big[\frac{8}{\alpha'}+R+4(\nabla\Phi)^2-f_1(T)(\nabla T)^2+f_2(T)+\cdots\Big]-\frac{\pi\alpha'}{2}f_3(T)(F^{(+)})^2-\frac{\pi\alpha'}{2}f_3(-T)(F^{(-)})^2+\cdots\Big\}. \tag{2.6}$$

The first few terms in a Taylor expansion of the f_i

[1]Unfortunately, it is standard in the literature to use Φ for both the Liouville superfield and the background dilaton.

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functions are

$$f_1(T) = \frac{1}{2} + \cdots, \qquad f_2(T) = \frac{1}{\alpha'}T^2 + \cdots,$$
$$f_3(T) = 1 - 2T + 2T^2 + \cdots. \tag{2.7}$$

There is evidence that the exact expression for $f_3(T)$ is e^{-2T} [5,6], and we will use this form for f_3 in our calculations.

III. AdS$_2$ SOLUTIONS

A. Equations of motion

To simplify the action (2.6), we will dualize the R-R field strengths:

$$-\frac{2\pi\alpha'}{4} f_3(\pm T)(F^{(\pm)})^2 {*1} = -\pi\alpha' f_3(\pm T) F^{(\pm)} \wedge {*F^{(\pm)}}$$
$$\to -\frac{1}{4\pi\alpha'} f_3(\mp T) q_\pm^2 {*1} + q_\pm F^{(\pm)}$$
$$\to -\frac{1}{4\pi\alpha'} q_\pm^2 f_3(\mp T) * 1.$$

In the second line, we have introduced an auxiliary field $q_\pm$. The equation of motion for $q_\pm$ is

$$q_\pm = -2\pi\alpha' f_3(\pm T) * F^{(\pm)}, \tag{3.1}$$

which, when substituted in, gives the original action. In the third line, we have integrated out $A^{(\pm)}$ which constrains $q_\pm$ to be a constant. Therefore, in the third line, the fields $A^{(\pm)}$ and $q_\pm$ are no longer functionally integrated. The full action can now be written as

$$S = \int dx dt \sqrt{-g} \left\{ \frac{e^{-2\Phi}}{2\kappa^2} \left[\frac{8}{\alpha'} + R + 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) + \cdots \right] - \frac{1}{4\pi\alpha'} f_3(-T) q_+^2 - \frac{1}{4\pi\alpha'} f_3(T) q_-^2 + \cdots \right\}. \tag{3.2}$$

Varying with respect to the metric $g_{\mu\nu}$, dilaton Φ, and tachyon T gives the equations of motion

$(\delta \mathbf{g})$

$$\frac{1}{2} g^{\mu\nu} \left\{ \frac{e^{-2\Phi}}{2\kappa^2} \left[\frac{8}{\alpha'} + 4\nabla^2\Phi - 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) \right] - \frac{1}{4\pi\alpha'} f_3(-T) q_+^2 - \frac{1}{4\pi\alpha'} f_3(T) q_-^2 \right\} + \frac{e^{-2\Phi}}{2\kappa^2} [-2\nabla^\mu\nabla^\nu\Phi + f_1(T)\nabla^\mu T \nabla^\nu T] = 0, \tag{3.3}$$

$(\delta\Phi)$

$$\frac{8}{\alpha'} + R + 4\nabla^2\Phi - 4(\nabla\Phi)^2 - f_1(T)(\nabla T)^2 + f_2(T) = 0, \tag{3.4}$$

and

$(\delta \mathbf{T})$

$$\frac{e^{-2\Phi}}{2\kappa^2} \left[2f_1(T)\nabla^2 T + f_1'(T)(\nabla T)^2 - 4f_1(T)(\nabla_\mu\Phi)(\nabla^\mu T) + f_2'(T) \right] - \frac{1}{4\pi\alpha'} f_3'(-T) q_+^2 - \frac{1}{4\pi\alpha'} f_3'(T) q_-^2 = 0, \tag{3.5}$$

where primes denote differentiation with respect to T. Setting Φ and T constant, we find

$(\delta \mathbf{g})$

$$\frac{e^{-2\Phi}}{2\kappa^2} \left[\frac{8}{\alpha'} + f_2(T) \right] - \frac{1}{4\pi\alpha'} q_+^2 f_3(-T) - \frac{1}{4\pi\alpha'} q_-^2 f_3(T) = 0, \tag{3.6}$$

$$(\delta\Phi) \qquad \frac{8}{\alpha'} + R + f_2(T) = 0, \tag{3.7}$$

and

$(\delta \mathbf{T})$

$$\frac{e^{-2\Phi}}{2\kappa^2} f_2'(T) - \frac{1}{4\pi\alpha'} q_+^2 f_3'(-T) - \frac{1}{4\pi\alpha'} q_-^2 f_3'(T) = 0. \tag{3.8}$$

With the AdS$_2$ metric

$$ds^2 = \frac{-4l^2}{\sin^2(u^+ - u^-)} du^+ du^-, \tag{3.9}$$

the Ricci scalar is

$$R = -2/l^2. \tag{3.10}$$

B. Solutions $T = 0$

The solution with $q_- = q_+ \equiv q$ and $T = 0$ satisfies the equations of motion with AdS radius given by

$$l^2 = \alpha'/4 \tag{3.11}$$

and dilaton given by

$$e^{-2\Phi} = \frac{\kappa^2}{8\pi} q^2. \tag{3.12}$$

This solution is related to the near-horizon geometry found in [7]. A notable feature of this solution is that the curvature radius is fixed at a value of order the string length. This implies that higher order α' terms in the effective action will be important. This will be addressed in Sec. IV. Also, note that we are free to tune the string coupling to zero by ramping up the strength of the R-R flux.

In this case, the "tachyon" is massive for all values of q. This can be seen as follows. The δT equation of motion, to first order in T, gives us

$$\left\{\nabla^2 + \nabla^2\Phi - (\nabla\Phi)^2 + \frac{2}{\alpha'} - \frac{4\kappa^2}{\pi\alpha'} e^{2\Phi} q^2\right\}(e^{-\Phi}T) = 0. \tag{3.13}$$

The $\delta\Phi$ equation of motion, to zero order in T, tells us that

$$\nabla^2\Phi - (\nabla\Phi)^2 + \frac{2}{\alpha'} = -\frac{R}{4}, \tag{3.14}$$

and, when substituted into the linearized δT equation, gives us

$$\left\{\nabla^2 - \frac{R}{4} - \frac{4\kappa^2}{\pi\alpha'} e^{2\Phi} q^2\right\}(e^{-\Phi}T) = 0. \tag{3.15}$$

Finally, substituting our background expressions for Φ and R, we get

$$\left(\nabla^2 - \frac{30}{\alpha'}\right)(e^{-\Phi}T) = 0, \tag{3.16}$$

so that the tachyon mass is $m_T^2 = \frac{30}{\alpha'} = \frac{15}{2l^2}$. The authors of [4] noted that, in ten dimensions, R-R flux could stabilize the tachyon potential. In our two-dimensional case, we see that the R-R flux makes the otherwise-massless tachyon massive.

Solutions to the wave equation in AdS_2 are most readily attained in Poincaré coordinates, in which

$$ds^2 = l^2 \frac{-dt^2 + dy^2}{y^2}. \tag{3.17}$$

In these coordinates, the wave equation takes the form

$$\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} - \frac{l^2 m_T^2}{y^2}\right)T(t, y) = 0. \tag{3.18}$$

Using separation of variables, we can write the general time-dependent, positive-frequency solution as $e^{-i\omega t}\chi(y)$. The normalizable solution is readily obtained in terms of a Bessel function as

$$T_w(t, y) = e^{-i\omega t}\sqrt{\frac{y}{2}} J_{h_\pm - 1/2}(\omega y), \tag{3.19}$$

where $h_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + 4l^2 m_T^2}$. The static solutions are obtained by noting that the wave equation

$$y^2 \frac{\partial^2}{\partial y^2} T = l^2 m_T^2 T \tag{3.20}$$

implies that $T \sim y^n$, where $n(n-1) = l^2 m_T^2$. Therefore, the general static solution is

$$T = a y^{h_+} + b y^{h_-}. \tag{3.21}$$

Note that, although these static solutions are non-normalizable, they may make an appearance as approximate solutions in regions of spacetime that are AdS-like.

Solutions to the wave equation in global coordinates are a little more difficult to come by, but they have been worked out in [8]. In the global coordinates

$$ds^2 = l^2 \frac{-d\tau^2 + d\sigma^2}{\cos^2\sigma}, \tag{3.22}$$

the normalized positive-frequency solutions are

$$T_n(\tau, \sigma) = \Gamma(h) 2^{h-1} \sqrt{\frac{n!}{\pi\Gamma(n+2h)}} e^{-i(n+h)\tau} (\cos\sigma)^h C_n^h(\sin\sigma), \tag{3.23}$$

where $n = 0, 1, 2, \ldots$, C_n^h is the Gegenbauer polynomial, and h is once again related to m_T^2 by $h(h-1) = l^2 m_T^2$. Note that, unlike in Poincaré coordinates, the spectrum in global coordinates is discrete.

C. Solutions with $T \neq 0$

The solution given in the previous section can be deformed by moving the constant value of T away from zero. The solution is given by

$$l^2 = \frac{\alpha'/4}{1 + \frac{\alpha'}{8} f_2(T)}, \tag{3.24}$$

$$e^{-2\Phi} = \frac{\frac{\kappa^2}{16\pi}[q_+^2 f_3(-T) + q_-^2 f_3(T)]}{1 + \frac{\alpha'}{8} f_2(T)}, \tag{3.25}$$

and

$$\frac{q_-^2}{q_+^2} = \frac{f_3(-T)}{f_3(T)} \frac{8/\alpha' + f_2(T) - f_2'(T)/2}{8/\alpha' + f_2(T) + f_2'(T)/2}. \tag{3.26}$$

Again, it is clear that, for all solutions in this family, we can send the string coupling to zero while holding fixed both the tachyon expectation value T and the AdS radius l. This is accomplished by sending q_-^2 and q_+^2 to infinity while holding the ratio q_-^2/q_+^2 fixed.

It is not evident from these equations whether or not there exists an AdS_2 solution with one of the q's, say q_-, set to zero. Setting $q_- = 0$ would require a T of order 1, but to understand such large values of T would require a more complete knowledge of f_2. Specifically, $q_- = 0$ would require that

$$\frac{8}{\alpha'} + f_2(T) - \frac{1}{2} f_2'(T) = 0, \tag{3.27}$$

and it is not known whether this equation has solutions.

IV. DISCUSSION

It should be noted that the AdS spaces presented here are solutions to the first few terms in the effective action. Since the AdS radius is of order the string length, we expect higher order terms in α' to change some of the quantitative features of the solutions, such as the exact value of the AdS radius or the true mass of the tachyon. However, as we shall discuss here, the qualitative features of the AdS solutions are rather generic and are not expected to be changed by the higher order α' terms.

We can ask what other terms might make contributions to the equations of motion, and, therefore, might change features of the AdS solution. For simplicity, let us concentrate on the $T = 0$ solution found in Sec. III C. Since we seek an AdS solution in which R, T, and Φ are constant and $F^{\pm}$ is nondynamical, the most general term of interest in the dualized action is $\alpha'^{n-1} e^{(2m-2)\Phi} T^p R^n q_{\pm}^{2m}$. The dilaton dependence is fixed by the number of R-R field strengths in the monomial [9,10]. Since we are considering the $T = 0$ solutions in this discussion, terms with $p > 1$ will not contribute to any of the field equations. The $p = 1$ terms will contribute to the δT variation, but the proposed symmetry [1] of the theory under $T \rightarrow -T$ and $q_+ \leftrightarrow q_-$ guarantees that the contribution to the equation of motion will be proportional to $(q_+^{2m} - q_-^{2m})$. Setting $q_+^2 = q_-^2 \equiv q^2$ as before, this term disappears from the field equations.

Having dispensed with terms involving T, we are left to focus on terms of the form $\alpha'^{n-1} e^{(2m-2)\Phi} R^n q^{2m}$. Under variation of the metric, the higher order terms in the action

$$\sum_{n,m} c_{n,m} \int \sqrt{-g} e^{(2m-2)\Phi} \alpha'^{n-1} R^n q^{2m}$$

modify the δg field equation to

$$\frac{e^{-2\Phi}}{2\kappa^2} \frac{8}{\alpha'} - \frac{q^2}{2\pi\alpha'} + \sum_{n,m} c_{n,m} (1-n) e^{(2m-2)\Phi} \alpha'^{n-1} R^n q^{2m} = 0.$$

We still seek a one-parameter family of solutions (with $T = 0$) in which q^2 is proportional to $e^{-2\Phi}$, and we will denote the constant of proportionality as B:

$$q^2 = B e^{-2\Phi}.$$

The δg equation of motion may now be written as

$$8 - \frac{\kappa^2}{\pi} B - 2\kappa^2 \sum c_{n,m} (n-1)(\alpha' R)^n B^m = 0.$$

Similarly, the $\delta\Phi$ equation of motion becomes

$$8 + \alpha' R - 2\kappa^2 \sum c_{n,m} (m-1)(\alpha' R)^n B^m = 0.$$

So long as there are simultaneous solutions to these two equations for some negative R and positive B, a one-parameter family of AdS solutions exists in which the string coupling may be tuned towards zero. This one-parameter family of AdS solutions, with Ricci scalar R, would be parametrized by q^2 with $e^{2\Phi} = B/q^2$.

The evidence gathered here suggests that the qualitative structure of the AdS solutions is rather generic and is likely to be unaffected by terms higher order in α'. This fact motivates a search for the corresponding world sheet sigma model describing type 0A strings propagating in these AdS_2 spaces. Because of the existence of nonzero R-R fluxes, the correct sigma model will most likely not be found using the Neveu-Schwarz-Ramond formalism. Fortunately, several other world sheet formalisms have been developed that have allowed for quantization of the string in R-R backgrounds. For example, the hybrid formalism has been used to study superstring quantization in $AdS_3 \times S_3$ [11], $AdS_2 \times S_2$ backgrounds [12], and curved 2D backgrounds [7].

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